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INVESTIGATING INFLATION DYNAMICS AND STRUCTURAL CHANGE WITH AN ADAPTIVE ARFIMA APPROACH

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Investigating Inflation Dynamics and Structural Change with an Adaptive ARFIMA Approach.

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Abstract

Previous models of monthly CPI inflation time series have focused on possible regime shifts, non-linearities and the feature of long memory. This paper proposes a new time series model, named *Adaptive* $ARFIMA$, which appears well suited to describe inflation and potentially other economic time series data. The *Adaptive* $ARFIMA$ model includes a time dependent intercept term which follows a Flexible Fourier Form. The model appears to be capable of succesfully dealing with various forms of breaks and discontinities in the conditional mean of a time series. Simulation evidence justifies estimation by approximate MLE and model specification through robust inference based on $QMLE$. The *Adaptive* $ARFIMA$ model when supplemented with conditional variance models is found to provide a good representation of the G7 monthly CPI inflation series.

Key words: *ARFIMA*, $FIGARCH$, long memory, structural change, inflation, G7.
JEL classification: C15 and C22

1 Introduction

The understanding of inflation dynamics has long attracted the attention of monetary and financial economists. Many recent articles have applied time series models with either long memory, non-linear switching, regime shifts and structural break analysis to inflation series. This paper, applying up to date statistical tests, finds evidence that the long memory feature appears to represent the total history and sub periods of the G7 monthly inflation series since the 1950s; it also finds additional non-linear effects in the data. Hence, G7 inflation series show both fractionally integrated behavior and generic non-linearity.

A new model, designated as the *Adaptive* $ARFIMA$, or $A-ARFIMA$ model, augmenting the traditional long memory model with a time dependent intercept term, evolving according to the Gallant's (1984) Flexible Fourier Form (FFF), is then introduced. The paper also discusses some theoretical issues concerning estimation and specification testing and finds that approximate MLE is found to be adequate from a detailed simulation study. Further simulation evidence also indicates that the model is capable of providing a good representation for sharper structural breaks which may occur as background "noise" within the slow long memory adjustment to shocks that are driving inflation.

The $A-ARFIMA$ model is then applied to the monthly CPI inflation series for the G7 countries. First, evidence is presented to document the apparent stylized fact of inflation having strongly dependent autocorrelation with apparent superimposed nonlinearities and changes in regimes due to policy changes with possibly abrupt structural breaks. The FFF type of time varying intercept coupled with the long memory $ARFIMA$ process is found to be an excellent description and competitor with more elaborate models.

The plan of the rest of this paper is as follows: the next section summarizes some of the previous literature and describes some of the more important nonlinear and long memory time series properties of monthly G7 inflation series for the G7 economies. The application of some relatively new tests indicates the presence of both nonlinear and long memory components in the monthly inflation series. Section 3 describes some econometric aspects of the new Adaptive $ARFIMA$ model that is proposed in this article. Section 3 also includes some Monte Carlo evidence that provides favorable evidence on the adequacy of the estimation method and also on the robustness of the model in accounting for process with long memory, breaks and non linearities. Section 4 of the paper then reports estimates of the model on the monthly G7 inflation series and section 5 then concluded with a review of the economic implications of the new results and new model.

2 Persistence and Long Memory Characteristics of Inflation

The properties of inflation and its response to shocks through impulse response weight analysis have been the subject of much previous research. Previous work by Ball and Ceccheti (1990) suggested a unit root in inflation; and Rose (1988) discussed the implications of this for the real rate of interest and its use in asset pricing models. Clearly, the concept of non stationary inflation does not seem consistent with the belief that central banks have been successful in using monetary policy for inflation targeting. However, deviations from a target may well be strongly persistent, as factors other than monetary policy can impact inflation.

Many studies such as Geweke and Porter Hudak (1982), Hassler and Wolters (1995), Baillie, Chung and Tieslau (1996) and Baum, Barkoulas and Caglayan (1999) have found evidence for long memory in the mean of inflation. On denoting the inflation process by y_t, then the series is defined to be $I(d)$ if

$$(1-L)^d y_t = u_t$$

where u_t is a short memory $I(0)$ process, and $-0.5 < d < 0.5$. The spectral density functions of y_t and u_t are represented by $f(\omega)$ and $f^*(\omega)$, respectively, so that

$$f(\omega) = |1-\exp(i\omega)|^{-2d} f^*(\omega).$$

However, the above studies can be legitimately criticized for failing to account for the nonlinearities, regime shifts due to policy changes and also the possibility of structural breaks. Neglecting these factors may give rise to the appearance of long memory, while in reality it is not the true data generating process. Brunner and Hess (1993) provided one of the first models of regime shifts in the mean and volatility of inflation; while Bos, Franses and Ooms (1999, 2001), Morana (2002), Hyung and Franses (2001), Bagliano and Morana (2006) have all provided different analyses of long memory and nonlinearities, including structural breaks in inflation. These studies are important since it is well known that neglecting of structural breaks will lead to an upward bias in subsequent estimates of persistence parameters. In particular, see Granger and Hyung (2004) and Diebold and Inoue (2001). Evidence in favor of the joint presence of breaks and long memory in the mean of inflation rates has been provided by Bos et al. (2001) and Morana (2002), for the US and the euro area, respectively. These results are consistent with breaks in inflation being due to changes in monetary policy regimes, which have occurred in many industrialized countries since WWII and particularly since the 1980s, when successful disinflation policies have apparently stabilized inflation to historically low levels.

In the following analysis, seasonally unadjusted CPI inflation data series for the G-7 countries of US, Japan, UK, Germany, France, Italy and Canada, from January 1948 through October 2006 are used. This gives a total of $T = 706$ observations. The Local Whittle semi parametric estimator of the fractional differencing parameter d, denoted by $\hat{d}_{LW}$, is obtained by minimizing the objective function

$$\ln\left[\frac{1}{m}\sum_{j=1}^{m}\omega_j^{2d} I(\omega_j)\right] - \frac{2d}{m}\sum_{j=1}^{m}\log(\omega_j) \tag{1}$$

with respect to d, where $I(\omega_j)$ is the periodogram given by $I(\omega_j) = \frac{1}{2\pi T}\left|\sum_{j=1}^{T} y_t e^{i\omega_j t}\right|^2$. Following Robinson (1995), the Local Whittle estimator of d is known to have the limiting distribution of

$$m^{1/2}\left(\hat{d}_{LW} - d_0\right) \rightarrow N\{\mathbf{0}, (\mathbf{1/4})\},$$

where d_0 denotes the true value of d, and m represents the bandwidth in equation (1) In all the applications in this study the estimator is based on the bandwidths of $m = T^i$, for $i = 0.5, 0.6, 0.75, 0.80$, and are reported in Table 1. Evidence of long memory can be detected for all the series, with Japan, Germany and France being the countries characterized by the lowest persistence with estimated long memory parameters slightly over 0.2, but statistically significant. Canada, US, UK and Italy have higher levels of persistence, which sometimes exceed the region of stationary long memory ($d > 0.5$) and may possibly be related to neglecting some forms of non linearity.

A major issue in the finding of apparent long memory has been possible existence of regime shifts and structural breaks. This can be partially investigated by the use of a test developed by Shimotsu (2006), which tests that the sub samples of the realization of a time series have the same long memory parameter as that of the complete series. The series is split into b blocks with T/b observations in each block. The Local Whittle estimate of the long memory parameter for the ith block is denoted by $\hat{d}_i$, then under the null hypothesis of $H_0 : d_0 = d_{1,0} = d_{2,0} = ... = d_{b,0}$, versus $H_1 : H_0$ is incorrect; where $d_{i,0}$ is the true value of d in the ith block. Then on defining,

$$\hat{d}_b^{/} = \begin{pmatrix} \hat{d} - d_0 & \hat{d}_1 - d_0 & . & . & \hat{d}_b - d_0 \end{pmatrix},$$

and

$$A = \begin{pmatrix} 1 & -1 & . & . & . & 0 \\ 0 & 1 & -1 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & 0 & . & . & .. & -1 \end{pmatrix} \text{ and } V = \begin{pmatrix} 1 & i_b^{/} \\ i_b & bI_b \end{pmatrix}$$

where I_b is the identity matrix of order b and i_b is a b dimensional column vector of ones. The Wald tests statistic for testing the $H_0 : d_0 = d_{1,0} = d_{2,0} = ... = d_{b,0}$, is then shown by Shimotsu (2006) to be,

$$W = 4m(A\hat{d}_b^{/})(AVA^{/})^{+}(A\hat{d}_b^{/}),$$

where $(AVA^{/})^{+}$ denotes the generalized inverse of $(AVA^{/})$. Then Shimotsu (2006) shows that W will have a limiting distribution of χ^2_{b-1}. Shimotsu (2006) also proposes an adjustment to this statistic that is motivated by the fact that the practical implementation of the Local Whittle procedure often finds the asymptotic variance of $\hat{d}_{LW}$ to exceed $(1/4m)$. Hence Shimotsu recommends the use of the adjusted Wald test statistic denoted by

$$W_c = 4m(c_{m/b})(b/m)(A\hat{d}_b^{/})(AVA^{/})^{+}(A\hat{d}_b^{/}),$$

where $c_m = \sum_{j=1}^{m} v_j^2$, and

$$v_j = \ln(\omega_j) - m^{-1}\sum_{j=1}^{m}\ln(\omega_j) = \ln(j) - m^{-1}\sum_{j=1}^{m}\ln(\omega_j)$$

and the test statistic W_c will also have an asymptotic χ^2_{b-1} distribution. The latter is the statistic used in empirical work in this paper.

On applying the test to the G7 inflation series from February 1948 through January 2006, $b = 4$ sub samples, with 174 observations each, can be investigated (Table 2). The Local Whittle estimator indicates the presence of statistically significant fractional differencing parameter for most of the seven countries and for most of the four sub periods. The only real exceptions are for Canada and France in the fourth period, dating from 1991 through 2006, for which an $I(0)$ inflation processes is found. However, in general there is considerable evidence for the presence of long memory in virtually all regimes and across all seven countries.

On considering $b = 4$ sub periods, and for a bandwidth of $T^{0.6}$, the null hypothesis of a constant long memory parameter across sub periods cannot be rejected for any of the $G7$ countries. This evidence is somewhat fragile, especially for Italy, where selection of different bandwidths can lead to a rejection of the null hypothesis.

3 Tests for Non-Linearity

Clearly neglecting non linear features may lead to an inappropriate finding of long memory and Ooms and Doornik (1999) have emphasized this in the context of ARCH; moreover, Baillie, Chung and Tieslau (1996) and Baillie, Han and Kwon (2002) have estimated various parametric models for long memory in the conditional mean of inflation, while simultaneously using various formulations of $GARCH$ models.

An appropriate test for the finding of long memory, being due to neglected nonlinearities, has been recently provided by Baillie and Kapetanios (2007). One test involves an Artificial Neural Networks test denoted by ANN, which requires filtering the series by the Local Whittle estimate of d to obtain an estimate of the short memory component,

$$\hat{u}_t = (1-L)^{\hat{d}} y_t \approx \sum_{i=1}^{t-1} \pi_i(\hat{d}) y_{t-i}. \tag{2}$$

The implementation of the test requires use of q constructed regressors

$$\phi(\sum_{i=1}^{p} \gamma_{ij} \hat{u}_{t-i}),\ j = 1, \ldots, q,$$

where $\tilde{q}$ is the largest principal component of the constructed regressors

$$\hat{u}_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \hat{u}_{t-i} + \sum_{j=1}^{\tilde{q}} \beta_j \tilde{\phi}_{j,t} + \epsilon_t, \tag{3}$$

where $\tilde{\phi}_{j,t}$ denotes the $(j+1)$ principal component. A standard LM test is then based on the test statistic TR^2, where R^2 is the unscented squared multiple correlation coefficient of a regression of $\hat{\varepsilon}_t$ on a constant, $\hat{u}_{t-i}$, $i = 1 \ldots, p$, $\tilde{\phi}_{j,t}$, $j = 1, \ldots, \tilde{q}$, and $\hat{\varepsilon}_t$ is the residual of the regression of $\hat{u}_t$ on a constant and $\hat{u}_{t-i}$, $i = 1 \ldots, p$. Under the null hypothesis, this test statistic has an asymptotic $\chi^2_{\tilde{q}}$ distribution.

An alternative test is the Logistic Neural Network (LNN) test, which Baillie and Kapetanios (2007) derive from an extension of the Terasvirta, Lin and Granger (1993) (TLG) test. The test approximates the LNN by a Taylor expansion and tests for the significance of these additional terms when they are subsequently substituted into the model. For example, Terasvirta, Lin and Granger (1993) suggest the use of a third order Taylor expansion, while Baillie and Kapetanios (2007) point out that a highly nonlinear data generating process may well require higher order terms, as for example a fourth order Taylor series expansion, i.e.

$$\hat{u}_t = \beta_0 + \sum_{i=1}^{p} \beta_i \hat{u}_{t-i} + \sum_{j=2}^{4} \sum_{i=1}^{p} \gamma_{0,i,j} \hat{u}^j_{t-i} + \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \gamma_{1,i,j} \hat{u}_{t-i} \hat{u}_{t-j} + \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \gamma_{2,i,j} \hat{u}^2_{t-i} \hat{u}^2_{t-j} +$$
$$\sum_{s=0}^{1} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \gamma_{3,s,i,j} \hat{u}^{2-s}_{t-i} \hat{u}^{s+1}_{t-j} + \sum_{s=0}^{1} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \gamma_{4,s,i,j} \hat{u}^{3-2s}_{t-i} \hat{u}^{2s+1}_{t-j} + \epsilon_t, \tag{4}$$

to be applied to the filtered series $\hat{u}_t$. For ease of computation, only cross products and powers of up to two lags may be considered. The restriction that the γ coefficients are all zero can then be tested by means of a Wald test. In what follows, the fourth order expansion has been employed, and the models underlying these tests are denoted as the TLG_i models, with $i = 2, 3, 4$.

Baillie and Kapetanios (2007) show that the TLG_i test statistics can also be based on simultaneous estimation in the time domain of the equations

$$(1-L)^d y_t = u_t$$

and

$$u_t(d) = \beta_0 + \sum_{i=1}^{p} \beta_i u_{t-i}^j(d) + \sum_{j=2}^{4}\sum_{i=1}^{p} \gamma_{0,i,j} u_{t-j}^j(d) + \sum_{i=1}^{p-1}\sum_{j=i+1}^{p} \gamma_{1,i,j} u_{t-i}(d) u_{t-j}(d) +$$
$$\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} \gamma_{2,i,j} u_{t-i}^2(d) u_{t-j}^2(d) + \sum_{s=0}^{1}\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} \gamma_{3,s,i,j} u_{t-i}^{2-s}(d) u_{t-j}^{s+1}(d) +$$
$$\sum_{s=0}^{1}\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} \gamma_{4,s,i,j} u_{t-i}^{3-2s}(d) u_{t-j}^{2s+1}(d) + \epsilon_t \tag{5}$$

where

$$u_t(d) = y_t - \sum_{l=0}^{t-p} \pi(d)_l y_{t-l} \approx y_t - \sum_{l=0}^{\infty} \pi(d)_l y_{t-l} = (1-L)^d y_t$$

and ϵ_t is assumed to be Gaussian although modifications to this assumption can be made through $QMLE$ procedure. The notation $u_t(d)$ is used to denote that the series is a function of d. This is in contrast to the final filtered series obtained from the approximate time domain MLE, which is denoted by $\hat{u}_t$.

The results of applying the ANN and TLG tests to the investigated inflation data are presented in Table 3, wher the p-value is given for all the test statistics. The results from applying both the ANN and TLG tests are broadly consistent for each series, with clear evidence of nonlinearity for the US, Japan and Germany, while some ambiguity can be found for Italy. For ease of comparison the results for the residuals from the various estimated models are also included and will be discussed in more detail later. Suffice to say that there is no evidence of remaining non-linearities from the residuals of the $A - ARFIMA$ model

4 The Adaptive ARFIMA Model

The above findings have indicated the presence of long memory in sub periods, as well as significant evidence for the presence of non-linearity over the whole sample realization. These findings are very much in accord with different components of the literature that have either found long memory or nonlinearity in the inflations series. Recently, Boero, Smith and Wallis (2007) have found evidence of structural breaks in UK inflation and prefer simple autoregressive models in mean and variance to the $AR(p) - ARCH$ specification originally preposed by Engle (1982). Osborn and Sensier (2008) find similar results for UK inflation with a break point in 1992 that coincides with the introduction of inflation targeting in the UK. Another related paper by Lanouar and Dominique (2008) finds evidence for the existence of a break points in the mean of US inflation that coincides with the Vietnam War and the two major oil price shocks in the 1970s. Most of these analyses involve use of the Bai (1997) and Bai and Perron (1998) tests for structural change.

The point of departure in this study is to propose a model which, while maintaining the slow decay of shocks to the inflation process, i.e. the so called long memory property, also allows for smooth forms of nonlinearity and potentially sharp breaks associated with discontinuities. Rather than spending resources on attempting to identify the exact date and magnitude of each break point, this paper proposes a Flexible Fourier Form representation for a time dependent intercept. The new *Adaptive ARFIMA* model is basically a regression with the conditional mean equation specified as a finite sum of harmonics, with a stationary long memory process disturbance, which is modeled as $ARFIMA(p, d, q)$ process. Hence, the observable variable y_t is specified as

$$y_t = \mu_t + u_t, \tag{6}$$

where the conditional mean is a trigonometric expansion or order k,

$$\mu_t = \mu + \sum_{j=1}^{k} [\gamma_j \sin(2\pi jt/T) + \delta_j \cos(2\pi jt/T)], \tag{7}$$

and the disturbance process is the $ARFIMA(p, d, q)$ process

$$\phi(L)\,(1-L)^d\, u_t = \theta(L)\epsilon_t, \tag{8}$$

where $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator of order p and q, respectively, with all their roots lying outside the unit circle, d is the fractional differencing parameter such that $-0.5 < d < 0.5$ and $\{\epsilon_t\}$ is a discrete time, real-valued, zero mean, serially uncorrelated, homoskedastic stochastic process. Hence, $E_{t-1}\epsilon_t = 0$ and $Var_{t-1}\epsilon_t = \sigma^2$. For the sake of simplifying notation, the above model is designated as an Adaptive $ARFIMA$, or $A - ARFIMA(p, d, q, k)$ process. Hence, the process is based on a regular $ARFIMA$ model with a time dependent intercept μ_t,which is represented by linear combination of harmonic terms. The choice of functional form for the time varying intercept is deliberately chosen to be a Flexible Fourier Form, or FFF, which has been proposed by Gallant (1984), and is well known to be an excellent approximation to a wide range of non linear functions, for suitable choice of k (see for instance Andersen and Bollerslev (1997) and Baillie and Morana (in press) for some forms of non stationarity in volatility, and Enders and Lee (2004) for the modelling of structural breaks in the mean process).

As mentioned previously, the great advantage of the above specification is that it does not require the joint modeling of long memory and structural change, and does not require any a priori assumptions concerning the timing of structural breaks.

Estimation and inference for the above $A - ARFIMA$ model can be carried out by means of the method of Quasi Maximum Likelihood Estimation ($QMLE$), where the Gaussian log likelihood

$$ln\{L(\boldsymbol{\vartheta}, u_1, ..., u_T)\} = -(0.5T)\ln(2\pi) - 0.5\sum_{t=1}^{T} \ln(\sigma^2) + u_t^2\sigma^{-2}$$

is numerically maximized with respect to the vector of the parameters

$$\boldsymbol{\vartheta} = (d, \boldsymbol{\phi}', \boldsymbol{\theta}', \mu, \boldsymbol{\delta}', \boldsymbol{\gamma}')',$$

which implements simultaneous estimation of all the model's parameters, including those in the flexible functional form, specifying the unconditional mean in the conditional mean process. Under fairly general conditions, the asymptotic distribution of the $QMLE$ is

$$T^{1/2}\left(\hat{\boldsymbol{\vartheta}}-\boldsymbol{\vartheta}_0\right)\rightarrow N\{\mathbf{0},\mathbf{A}(\boldsymbol{\vartheta}_0)^{-1}\mathbf{B}(\boldsymbol{\vartheta}_0)\mathbf{A}(\boldsymbol{\vartheta}_0)^{-1}\},$$

where $\boldsymbol{\vartheta}_0$ denotes the true value of the vector of parameters, and where $\mathbf{A}(\boldsymbol{\vartheta}_0)$ is the Hessian and $\mathbf{B}(\boldsymbol{\vartheta}_0)$ is the outer product gradient, both of which are evaluated at the true parameter values. Dahlhaus (1999,1989) and Mohering (1990) have shown that the same asymptotic properties hold also when the unconditional mean is not known and have to be estimated. In this latter case $T^{1/2}$ consistency of the Maximum Likelihood estimator is achieved for all the parameters, apart from the unconditional mean parameter, which is just $T^{1/2-d}$ consistent. Previous studies by Taqqu (1975) and Yajima (1988) have noted the slow rate of convergence of $T^{1/2-d}$ for the sample mean estimator in the case of stationary long memory of order d, rather than the usual $T^{1/2}$ rate found for the *i.i.d.* case. Simulation results of Baillie, Chung and Tieslau (1996) and Baillie, Han and Kwon (2002) point that the $QMLE$ estimator has a very satisfactory performance in terms of bias and RMSE for all the parameters, also when the innovations are not homoskedastic, for medium sized samples, i.e T greater than 500 observations.

Finally, Robinson (2006) has proved consistency and asymptotic normality of the cumulative sum of squares (CSS) estimator, which is usually employed to implement *QMLE* estimation in the *ARFIMA* framework. Similar properties can be expected for the parameters of the trigonometric component in the adaptive generalization of the $ARFIMA$ model.

4.1 Extension: the Adaptive *ARFIMA(p,d,q,k)-FIGARCH(1,b,1,h)* process

A simple generalization of the *A-ARFIMA* process is represented by the adaptive *ARFIMA-FIGARCH* process, or A_2*-ARFIMA-FIGARCH* process. This latter process, in addition to allow for long memory and structural change in the conditional mean, also allows for long memory and structural change in the conditional variance. In this framework the conditional variance process is allowed to follow the adaptive *FIGARCH* or *A-FIGARCH,* process of Baillie and Morana (in press).

The generalization is therefore achieved by defining the discrete time, real-valued, zero mean, serially uncorrelated stochastic process $\{\epsilon_t\}$ as $\epsilon_t\equiv z_t\sigma_t$, where $E_{t-1}[z_t]=0$ and $Var_{t-1}[z_t]=1$, and σ_t is a positive, time-varying measurable function with respect to the information set available at time $t-1$, which is denoted as Ω_{t-1}. Hence, σ_t^2 is the time dependent conditional variance defined as $\sigma_t^2=Var_{t-1}(u_t^2)=Var(u_t^2|\Omega_{t-1})$. Following Baillie and Morana (in press), the latter is expressed as the *A-FIGARCH(1,b,1,h)* process

$$[1-\beta L]\left(\sigma_t^2-w_t\right)=[1-\beta L-\psi L(1-L)^b]\epsilon_t^2, \tag{9}$$

where

$$w_t=w_0+\sum_{j=1}^{h}[\zeta_j\sin(2\pi jt/T)+\eta_j\cos(2\pi jt/T)], \tag{10}$$

and b is the long memory, fractional differencing parameter for the conditional variance, allowed to be in the interval $0<d<1$, w_t is the time-varying intercept following the Gallant (1984) functional form of order h, $\psi(L)=(1-\alpha L-\beta L)(1-L)^{-b}$. The polynomials $\psi(L)$ and $(1-\beta L)$ are assumed to have all their roots lying outside the unit circle.

On rearranging, it is found

$$\begin{aligned}\sigma_t^2 &= w_t + \left[1-(1-\beta L)^{-1}(1-L)^b(1-\psi L)\right]\epsilon_t^2 \\ &= \omega_t + \lambda(L)\epsilon_t^2,\end{aligned} \tag{11}$$

with $\lambda_0 = 1$, $\lambda_1 = d+\psi-\beta$, and, following Conrad and Haag (2006), $\lambda_i = \beta\lambda_{i-1}+(f_i-\psi)(-g_{i-1})$ $i > 1$, where $f_j = (j-1-d)/j$, for $j = 1, 2, ...$ and $g_j = f_j \cdot g_{j-1}$. As pointed out in Baillie and Morana (in press), non negativity constraints can be enforced in several ways. For instance, Conrad and Haag (2006) have recently proposed alternative and less restrictive forms, and they show for the case of $0 < \beta < 1$, either $\lambda_1 \geq 0$ and $\psi \leq f_2$ or $\lambda_{j-1} \geq 0$ and $f_{j-1} < \psi \leq f_j$ with $j > 2$; while for the case of $-1 < \beta < 0$, either $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ and $\psi \leq f_2(\beta+f_3)/(\beta+f_2)$ or $\lambda_{j-1} \geq 0$, $\lambda_{j-2} \geq 0$ and $f_{j-2}(\beta+f_{j-1})/(\beta+f_{j-2}) < \psi \leq f_{j-1}(\beta+f_j)/(\beta+f_{j-1})$ with $j > 3$. Similar restrictions hold for the *A-FIGARCH* model.

The consistency and asymptotic normality of the $QMLE$ estimator of the adaptive *FIGARCH* component of the model can be conjectured on the basis of available results from the estimation of $ARCH(\infty)$ processes. We remaind to Baillie and Morana (in press) for additional details.

5 Simulation Evidence

A relatively detailed simulation experiment was implemented to consider aspects of estimation and inference in the $A-ARFIMA$ model. The simulation is intended to (i) investigate the effectiveness of standard MLE for parameter estimation, (ii) to assess the usefulness of the model in capturing nonlinearity, structural change, and (iii) to compare with the standard $ARFIMA$ model.

In the subsequent experiments the $ARFIMA(p,d,q)$ model in equations (1) and (2) were generated with the three different designs of $(p=0, q=0)$, $(p=1, q=0)$ and $(p=0, q=1)$ with $\phi = (0, 0.15, 0.30)$, $\theta = (0, 0.15, 0.30)$, and $\theta = (0.15, 0.30, 0.45)$, with $\varepsilon_t \sim NID(0,1)$. The possibly time dependent intercept μ_t was generated from the three different designs of:

Design 1 has a constant intercept of $\mu_t = \mu = 0.5$, and corresponds to the standard case without structural breaks in the conditional mean.

Design 2 has a step change in the intercept at the midpoint of the sample, where the intercept is doubled at this point. Hence,

$$\mu_t = \begin{cases} 0.5 & t = 1, ..., T/2 \\ 1 & t = T/2+1, ..., T. \end{cases}.$$

Design 3 has two step changes equally spaced throughout the sample where the intercept increases eight fold, one third of the way through the sample and then decreases four fold after two thirds of the sample. Hence,

$$\mu_t = \begin{cases} 0.5 & t = 1, ..., T/3 \\ 4 & t = T/3+1, ..., 2T/3 \\ 1 & t = 2T/3+1, ..., T. \end{cases}.$$

Clearly, the estimation of the $A-ARFIMA$ model should prove superfluous in design 1, while the interest in designs 2 and 3 centers on the performance of $QMLE$ when the pure $ARFIMA$ and the $A-ARFIMA$ processes are estimated in the presence of structural breaks in the intercept of the conditional mean.

The $A-ARFIMA$ models are estimated for each design with one to four pairs of trigonometric terms included, i.e. $k = (1, 2, 3, 4)$. The number of simulated observations for each design is 5,000 observations, which includes the discarded first 3,500 or 4,500, leaving with simulated processes of sample size equal to 1,500 observations and 500 observations, respectively. In the CSS estimation the first 100 observations have been retained for initialization, and 1000 Monte Carlo replications have been employed in all of the designs. Tables 3 through 5 present the bias and root mean square error ($RMSE$) and the standard error (s.e.) of the estimators for both the $ARFIMA$ and $A-ARFIMA$ models. The following information can be derived from these tables:

(i) Neglecting structural breaks appears to lead to an upward bias in the estimated fractional differencing parameter in the pure $ARFIMA$ model. The magnitude of the bias is positively related to the complexity of the structural break process and also inversely related to the persistence, as measured by the size of the fractional differencing parameter.

(ii) The absence of a structural break in Design 1 is associated with the order of the trigonometric expansion (k) leading to an increase in the bias of the fractional differencing parameter. While designs with structural breaks lead to the bias in the fractional differencing parameter decreasing as k increases. This is particularly evident with the bias in the parameter estimates of the $A-ARFIMA$ model always being less than the corresponding biases for the pure $ARFIMA$ except for Design 1. For instance, for the $d = 0.15$ case, the bias in estimating d in the pure $ARFIMA$ model is 0.286 and 0.235 for designs 2 and 3 respectively, while the corresponding bias for the $A-ARFIMA$ model with $k = 4$ has biases of only -0.060 and 0.123 respectively. This favorable comparison for the $A-ARFIMA$ model also holds for the intermediate and strong long memory cases, and excellent estimates can be obtained by using a low order of $k = 1$ or $k = 2$.

(iii) For the weak long memory case, the inclusion of too many trigonometric components can lead to some bias in the estimated d parameter.

(iv) The standard error and RMSE of the estimated d parameter tends to increase with the value of k, but decreases with the complexity of the break process *and the sample size*. The latter finding also holds for the pure *ARFIMA* model. In general the RMSE is lower for the *ARFIMA* model than for the *A-ARFIMA* model for design one, while the *A-ARFIMA* model is superior to the *ARFIMA* model for designs two and three.

(v) The inclusion of short memory *ARMA* components and neglecting structural change leads to an upward bias in the estimates of d and the $MA(1)$ parameter and a downward bias in the estimate of the $AR(1)$ parameter. The bias tends to decrease as the value of d and as the sample size increases. Consistent with the *ARMA*(0,0) case, the unnecessary inclusion of trigonometric components, when $k = 0$ can lead to downward bias in the estimate of d. However, and importantly, in the presence of structural change, virtually unbiased estimates of d and the short memory parameters can be obtained by the selection of a suitable order for the trigonometric expansion. When the structure of the break process is sufficiently complex, as in design three, or when the sample size is large ($T = 1500$), there is always a benefit from modelling structural change[1]. The reduction in bias is dramatic, and is of the order of 80% for $T = 500$ and 30% for $T = 1500$; keeping the order of the trigonometric expansion as a first or second order, also leads to a significant reduction in bias of the $AR(1)$ parameter estimate.

(vi) Overall the Monte Carlo evidence indicates clear cut superiority of the *A-ARFIMA* model over the pure *ARFIMA* model in the presence of structural breaks. While the pure *ARFIMA* model is only slightly superior to the *A-ARFIMA* model in the presence of structural

[1] And at least in the $MA(1)$ case for T=500.

stability. Hence the benefits of modeling structural change seems to outweigh the cost of neglecting structural instability, although the appropriate selection of the order of the trigonometric expansion is key for the performance of the adaptive approach.

5.1 Modeling nonlinearity and long memory in inflation

The first set of models belong to the *ARMA-GARCH* family, where structural breaks and long memory in the mean and variance are neglected; the second set of models are for the *ARFIMA-FIGARCH* models, allowing for long memory but not for structural breaks; the third set of models belongs to the new class of adaptive long memory models, i.e the A_2-*ARFIMA-FIGARCH* class, allowing for both features.

The estimation results are reported in Tables 6 through 8. According to the BIC, the A_2-*ARFIMA-FIGARCH* is the preferred model in all cases, while the *ARFIMA-FIGARCH* model is always preferred to the *ARMA-GARCH* model. This is consistent with the finding that a long memory model may mimic, under certain conditions, also the properties of a short or long memory model subject to structural change. However, the selection of the A_2-*ARFIMA-FIGARCH* allows for a more accurate characterization of the data generating process of inflation.

After accounting for long memory, structural breaks and $GARCH$ effects, no additional evidence of non linearity can be found in G-7 inflation rates. These results are displayed in the last rows of table 3. Interestingly, persistence of the conditional mean of inflation is sizably lower than previously found in the literature, with the fractional differencing parameter taking values below 0.20 for most of the series considered.

As expected, the estimation of *ARMA-GARCH* models tends to lead to profligate specifications with the persistence parameter for the conditional mean being in the range 0.674 through 0.874 for five of the G7 countries. Also, as expected, allowing for long memory leads to the selection of far more parsimonious models in all of the cases.

Even more parsimonious specifications in terms of short memory dynamics, and an impact on the estimated value of d, are found by accounting for both long memory and structural change. In particular the persistence parameters estimated by the adaptive long memory models are in the range 0.110 (Japan) through 0.545 (France), with most of the estimates taking values below 0.20. There are also some reductions on the estimated long memory parameter for the conditional variance process once structural change is taken into account. The adaptive long memory model is generally the best fitting model according to the BIC information criterion.

6 Conclusions

This paper has addressed some of the issues in understanding the persistence, non-linearities and dynamics of the G7 inflation series. The paper has introduced the new adaptive *ARFIMA* process, or *A-ARFIMA* process, which generalizes the *ARFIMA* process by allowing for both long memory and structural change in either or both the conditional mean and conditional variance equations. The results of the analysis in this paper indicates the presence of long memory and structural change not only in the conditional mean dynamics, but also in the conditional variance dynamics of G-7 inflation rates. Interestingly, once accounted for structural breaks and conditional heteroskedasticity, no evidence of non linearity is left in the inflation rate series. Moreover, persistence in the conditional mean of inflation is found to be sizably lower than what previously found in the literature, with the fractional differencing parameter taking values below 0.20 for most of the series considered.

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